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Numerical simulation of the scaling behaviour in the negative Hall effect of high-T_c superconductors

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Abstract. Using the Wang–Ting model and including the influence of the thermal fluctuation on the flux motion, we have performed numerical simulation of the scaling behaviour in the negative Hall resistivity region of the high-T_c superconductors for the first time. A scaling law with an exponent $\beta \sim 1.7$ at low magnetic fields has been found in our simulation for the YBCO system, which is in good agreement with the experimental observation.

The behaviour of Hall resistivity in the mixed state of high-T_c superconductors is one of the most intriguing features in understanding flux dynamics, and stimulates a great deal of interest. The sign reversal of Hall resistivity ρ_{xy} in low magnetic fields and at temperatures close to but below the superconducting transition temperature T_c has been observed in many high-T_c materials [1–7], as well as in some low-temperature superconductors [8]. Moreover, the scaling behaviour of the positive Hall resistivity versus the longitudinal one has been found for both the temperature dependence in the TAFF region and the current dependence in the non-linear region: $\rho_{xy} \sim \rho_{xx}^{\beta}$ with $\beta \sim 2$ [9, 10]. Besides, in the region with the negative Hall resistivity, a scaling behaviour with $\beta \approx 1.7$ as a function of the temperature in YBCO film was reported by Luo *et al* [6]. How to satisfactorily explain the puzzling scaling behaviour and the sign reversal of Hall resistivity ρ_{xy} is one of the hottest problems in this field and quite challenging. On one hand, there are several models proposed to explain the sign reversal [4, 11–13]. Within the approximation of neglecting the thermal fluctuation Wang and Ting (WT) [11] had developed a theory in the framework of the Bardeen–Stephen (BS) [14] and Nozières–Vinen (NV) [15] approaches by taking into account the backflow current due to the pinning force. The obtained analytical results are qualitatively consistent with the experimental observations of negative Hall resistivity at low magnetic fields. In contrast to the WT model, Ferrell [12] suggested that the thermally excited quasiparticles colliding quasielastically with the hydrodynamic superfluid velocity field *outside* a vortex core, transfer momentum to the circulating velocity field, and such a process causes a sign of Hall angle opposite to that in the normal state. On the other hand, there exist two models to explain the scaling law for the Hall effect in the mixed state [16, 17]. Dorsey and Fisher (DF) [16] hypothesized that the vortex–glass transition exists in the mixed state of a 3D vortex system with a random disorder, and attributed the scaling behaviour to a kind of general glassy scaling in the vicinity of the vortex–glass transition. An appropriate $\beta \sim 1.7$ was produced by choosing the particle–hole asymmetry exponent $\lambda_v \sim 3$. Its demerit is that the observed scaling behaviour regime has to be restricted in a specific region near the vortex–glass transition temperature T_g. However, the probed temperature region in Luo *et*

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al's experiment [6] seems to be far from the T_g. In addition, using DF theory it is difficult to explain the scaling behaviour observed in the BSCCO or TlBaCaCuO samples with larger anisotropy and closer to a 2D system because the vortex–glass transition does not exist in a two-dimensional (2D) system at finite temperature. Recently, an alternative model has been proposed by Vinokur and co-workers [17]. They consider the scaling behaviour should be a general feature of any vortex state with disorder dominated dynamics, and derived a scaling relation $\rho_{xy} \sim \alpha\rho_{xx}^{\beta}$ with β being exactly equal to two and α as an *ad hoc* parameter. It was claimed that the exponent β is universal and independent of disorder and other parameters such as the temperature, the external magnetic field, and the applied current density. Their results seem to be in agreement with some experiments performed in the TAFF region within a certain range of the magnetic field [9, 10]. However, the parameter α appearing in their force balance equation ((1a) in their paper) was put into the model by hand and assumed to be independent of the pinning and the flux motion velocity in the TAFF and creep region. In fact, in the presence of pinnings, α should in general depend on pinnings and/or the velocity, since the flux has a 'normal core' and there exists a backflow current inside the core due to the pinning. In particular, α could vary significantly in different temperature regions (including the case with the sign change) [18]. Thus, their model seems to be rather artificial. More importantly, neither model mentioned above is able to explain the sign reversal of ρ_{xy}.

Very recently, based upon the BS normal core model and by taking into account the effect of both the thermal fluctuation and backflow in the core, Wang and co-workers [18] have proposed a unified theory for the mixed state Hall effect in type II superconductors, in which both the scaling behaviour with $\beta \sim 2$ and the sign reversal of ρ_{xy} can be naturally explained for the first time. The analytical results derived there agree qualitatively with most essential features found in experiments on Hall resistivity in the mixed state. Although no analytical analysis for the scaling behaviour of the negative Hall resistivity is given in the theory, a basic equation to describe both the Hall and the longitudinal resistivities is presented so that numerical calculations could be made for different magnetic fields and temperature regimes in the presence of thermal fluctuations and random pinnings, which makes it possible to compare quantitatively (or at least, semiquantitatively) the theoretical results with the experimental observation, especially for the negative Hall resistivity region.

Notice that the negative ρ_{xy} has been observed in many experiments for different high-T_c superconducting materials (YBCO, TlBaCaCuO, BSCCO), but the scaling behaviour for the negative Hall resistivity with $\beta \sim 1.7$ has only been observed by Luo *et al* in the YBCO system [6]. One could naturally raise the following questions. (1) Does the scaling law still hold in this negative Hall region? (2) Is the exponent $\beta = 2$ really universal? For example, with the temperature increasing further from the TAFF region the system will undergo a sign reversal of ρ_{xy} from positive to negative and go into the negative ρ_{xy} region. At that moment does the β value also undergo a change or not? Is it still equal to two? In this paper, we will investigate these crucial problems by using the numerical simulation method due to the absence of an analytical solution to the equation of flux motion with the random pinnings and thermal fluctuations.

In the negative Hall resistivity regime of the high-T_c superconductors, the temperature is not too far below T_c and the pinning potential can be comparatively low so that the thermal fluctuation will play a crucial role in assisting the motion of a flux within this region. In the presence of thermal fluctuations, the force balance equation for a single flux line can be derived as [11, 18, 19]

$$F + f_{\mathrm{drag}} + F_{\mathrm{t}} + F_{\mathrm{p}} = 0 \tag{1}$$

where F_t and F_p are respectively thermal noise and pinning forces acting on the flux, $F = Ne(v_T - v_L) \times \Phi_0$ is the Magnus force with $Nev_t = J$ as the applied current along the x-direction, and Φ_0 is the flux quantum in the direction $\hat{n}$ ($\perp$ ab plane). f_{drag} is the drag force which has the following form [11, 18–20]

$$f_{\text{drag}} = Nev_L \times \Phi_0 - \eta v_L + \Phi_0 \beta_0 (1 - \bar{\gamma}) J - \beta_0 (1 + \bar{\gamma}) F_p \times \hat{n} \tag{2}$$

where v_L is the velocity of the flux line, $\beta_0 = \mu_m H_{c2}$ with $\mu_m = \tau e/m$ as the mobility of the charge carrier and $H_{c2} = \Phi_0/2\pi\xi^2$ being the usual upper critical field with ξ as the superconducting coherence length, and $\eta = Ne\Phi_0\beta_0 = \Phi_0 H_{c2}/\rho_n$ is the usual viscous coefficient with ρ_n as the resistivity of the normal state. $\bar{\gamma} = \gamma(1 - \overline{H}/H_{c2})$ with $\overline{H}$ as the average magnetic field over the core and γ as the parameter describing contact force on the surface of the core, which depends on T in the following way [11]: $\gamma \sim 0$ (NV limit) for $\xi/l \ll 1$ and $\gamma \sim 1$ (BS limit) for $\xi/l \geqslant 1$ with l as the mean free path of the carrier. In detail, we rewrite equation (1) as

$$\eta v_L = F_L + F_p + F_t - \beta_0(1 - \bar{\gamma}) F_L \times \hat{n} - \beta_0 (1 + \bar{\gamma}) F_p \times \hat{n} \tag{3}$$

where $F_L = J \times \Phi_0$ is the Lorentz force. Equation (3) is rigorously derived in terms of the well known normal core model, and the transverse term $F_p \times \hat{n}$ is produced by the backflow current inside the normal core, which constitutes the essential physics of WT theory. In particular, it is worthwhile to mention that, in contrast with an argument in [17, 21], the term $\propto F_L \times n$ in equation (3) is believed not to to be in disagreement with any fundamental law because (i) in the effective force balance state, equation (3) can equivalently be expressed in different forms, which may contain a term $\propto F_L \times n$ or a term $v_\phi \times n$. Actually, there is no difference in final results for different forms. A simple but typical justification is that in terms of equation (3) which contains term $F_L \times n$, in the absence of pinnings and thermal noise, we are able to immediately recover BS ($\gamma = 1$) and NV ($\gamma = 0$) results which have been well accepted for many years. (ii) If the flux is completely pinned, which is the case discussed in [17] to support their argument, the total current inside the core is equal to zero in our framework (i.e. $J_{\text{tot}}^{\text{in}} = J_t^{\text{in}} + J_b^{\text{in}}$) [11], and thus there is no dissipation which stems from the in-normal core scattering of charge carriers with lattices. In view of equation (3), the point can also be understood by the fact that the total effective force (the terms on the RHS of equation (3)) acting on a flux from the superfluid, vanishes in the cases both $\parallel$ and $\perp$ J, thus the net force on the superfluid from a pinned vortex is zero as well.

So far, we have had a basic equation like equation (3) to describe the flux motion in the presence of thermal fluctuations and pinning. The x and y components of the equation can be written as

$$\begin{aligned} \eta v_{Lx} &= \beta_0 \Phi_0 (1 - \bar{\gamma}) J + F_{px} + F_t^x - \beta_0 (1 + \bar{\gamma}) F_{py} \\ \eta v_{Ly} &= -\Phi_0 J + F_{py} + F_t^y + \beta_0 (1 + \bar{\gamma}) F_{px}. \end{aligned} \tag{4}$$

It is non-trivial, in general, to solve v_{Lx} and v_{Ly} in equation (4). Although, as shown in [18], it is unnecessary to solve equation (4) in detail to show qualitatively the scaling behaviour of the positive ρ_{xy} as well as the sign reversal of Hall resistivity, in order to analyse the scaling behaviour in different temperature regions and to make comparison with the experimental observations, it is important to solve equation (4) numerically.

Because of the inhomogeneities in the materials, the pinning potential is always a random potential (here, the magnetic field $H \perp ab$ plane). The vortex mobility should be determined by the combined effects of both the random pinning potential and the random thermal fluctuations. As usual, F_p is considered to be produced by interaction of the vortex with a number of pinning centres randomly positioned at R_i and the individual pinning wells are chosen as the Gaussian form. Thus the following pinning potential is employed in the simulation:

$$U_p(r) = A_p \sum_i \exp(-|r - R_i|^2/\xi_{ab}^2) \tag{5}$$

where the amplitude A_p is the effective condensation energy stored in the vortex core per unit length, $A_p = (H_c^2/8\pi)(1-b)\xi_{ab}^2$ [22]. Here, ξ_{ab} represents the Ginzburg–Landau coherence length in the ab plane and $b = H/H_{c2}(T)$. Following the algorithm developed by Brass and Jensen [22], we can use the same algorithm to solve numerically equation (4) in a two-dimensional system having 400 pinning sites and a side length of 40 times the coherence length ξ with a periodic boundary condition. Notice that the parameter γ is argued as a rapidly varying function of temperature around a characteristic temperature T_0 [11]. In order to simulate this behaviour, it is convenient to suppose here that $\gamma = (e^{\alpha_0(t_0-t)}+1)^{-1}$ with $\alpha_0 \gg 1$, where $t_0 = T_0/T_c$ and $t = T/T_c$ are reduced temperatures. As usual, for YBCO material, we take $\xi_{ab}(0) = 27$ Å, $H_{c2}(0) = 127$ T, $H_c(0) = 2.72$ T; $\xi_{ab}(t) = \xi_{ab}(0)(1-t^2)^{-1/2}$, $H_c(t) = H_c(0)(1-t^2)$ and $H_{c2}(t) = H_{c2}(0)(1-t^2)$ in our simulation. Other parameters are chosen to be reasonable values: $T_0 = 65$ K, $\alpha_0 = 30$, because the precise values are unimportant for the scaling.

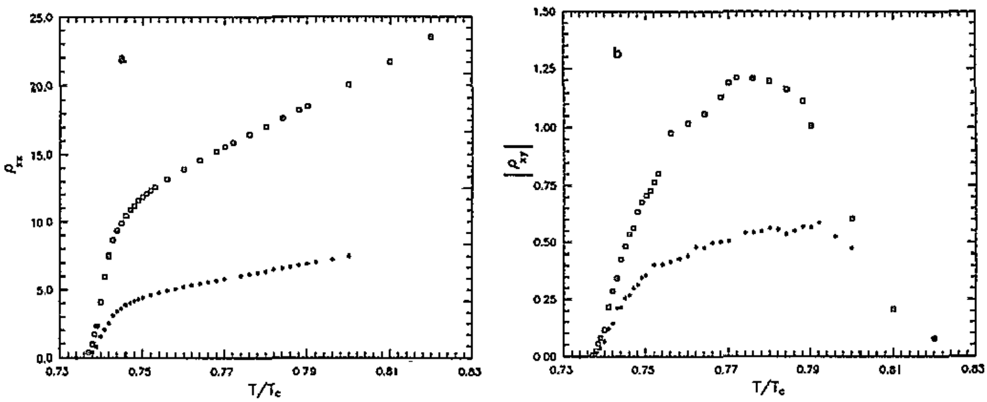

Figure 1. Longitudinal (a) and Hall (b) resistivity against temperature for different magnetic fields 1.5 T and 4 T; ∗——, 1.5 T; □——, 4 T.

The calculated results for ρ_{xx}, ρ_{xy} and the scaling behaviour are shown in figures 1 and 2. From these figures we can arrive at the following points:

(1) The temperature dependence of ρ_{xy} is qualitatively consistent with experimental observations, i.e. existence of a negative minimum at a certain temperature. The magnetic field dependence of ρ_{xy} exhibits monotonically increasing behaviour with increasing field below 4 T [23]. Although we did not carry out our simulation on ρ_{xy} for higher fields we may expect that its field dependence will have a maximum value due to decreasing of the pinning potential with increasing field, so reducing the negative Hall resistivity in our

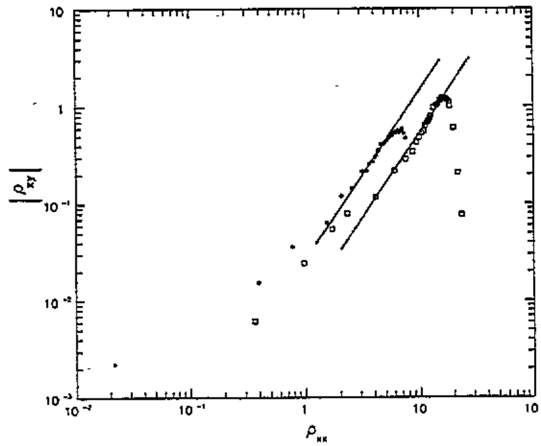

Figure 2. Log–log plot of $|\rho_{xy}|$ against ρ_{xx} at two magnetic fields 1.5 T and 4 T. Solid lines highlight the scaling behaviour $\rho_{xy} \propto \rho_{xx}^{\beta}$ with the scaling exponent $\beta \sim 1.7 \pm 0.1$. ∗——, 1.5 T; □——, 4 T.

framework, which seems not to be inconsistent with the experimental facts in YBCO samples [6].

(2) From $|\rho_{xy}|$ against ρ_{xx} on a log–log plot at fixed field 1.5 T and 4 T shown in figure 2, a power law relationship in this negative Hall resistivity region is clearly demonstrated. The scaling exponent $\beta \sim 1.7$ can be found from the data for both fields. Also, the region showing the scaling behaviour becomes wider for higher magnetic field, which is qualitatively consistent with Luo *et al*'s experiment [6]. However, it should be pointed out that the scaling lines have been collapsed to two distinct scaling curves with slightly different scaling exponents which is more obvious for higher fields. The portion in the lower-temperature region being much closer to T_g has a smaller exponent. This phenomenon is different from that in the TAFF region where no such kind of break has been observed. At the present time, because there is only one experimental data point for the scaling behaviour in the negative Hall effect (i.e. Luo *et al*'s result [6]) we are unable to discuss further this collapse of the scaling curve. However, if we look carefully at Luo *et al*'s result (figure 3 in their paper) [6] it seems that the break also exists in their observation, and could be identified (the scale of the ordinate in their figure 3 is too large to make this identification). As it is shown obviously in figure 2 that the collapse point is very close to the glass transition temperature T_g (about 1 K from T_g), we may conjecture that it may be caused by the significant fluctuation near the glass phase transition.

At this stage, we can conclude the following. (i) The WT model including a thermal noise term can give negative Hall resistivity at lower magnetic fields in a certain range of temperature, and its temperature dependence is rather well consistent with the experimental observations. More importantly, the scaling behaviour of negative ρ_{xy} can also be obtained in the WT model, which provides a strong support to the viewpoint that the effect of the backflow current on Hall resistivity due to pinning is crucial. It is noted that there has so far been only one model [18] able to give a unified explanation of both the negative Hall effect and the scaling behaviour. (ii) The exponent β obtained in our simulation on negative Hall effect is equal to $\sim 1.7 \pm 0.1$, which is in good agreement with Luo *et al*'s experimental value ($\sim 1.7 \pm 0.2$). It may demonstrate that the scaling relation is a universal phenomenon over different temperature regions, from TAFF, flux creep to flux flow, which may be mainly caused by the time reversal symmetry of the pinning potential [17, 18]. However, with increasing temperature, the temperature dependence of α will play a more and more important role in the scaling behaviour and cannot be neglected any longer, which may cause deviation of β from the value of two and produce the collapse of the scaling curve. Thus, in the creep region the value of β should not be a universal constant, but a sample dependent constant. It could be equal to different values smaller than two or larger than two.

Finally, it is worth pointing out that the relation $\langle F_{\mathrm{pin},i}\rangle = -\Gamma(v)v_{\mathrm{L}}$ [17, 18] is not assumed in the present simulation, which is one of key points in the analytical derivation [17, 18] of the scaling relation. It is known that analytical determination of the average pinning force is a highly non-trivial issue, which is also true in numerical simulations as well although maybe it is easier than that in the analytical method. More precisely numerical determination of the average pinning force is not the task of this paper and will be left for future investigation.

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[20] For simplicity, $\Phi_0/2\pi a^2$ (a, the radius of the flux core) is extrapolated to $H_{c2} - \overline{H}/2$ because it approaches H_{c2} when $\overline{H} \ll H_{c2}$, while it approaches $H_{c2}/2$ when $\overline{H} \to H_{c2}$ [14, 11]; in addition, a minor correction in connection with a small term $(\overline{H}/H_{c2})$ is made in the present form.
[21] There is a misunderstanding in the argument, because the authors seem to consider a pinned case without including the influence of pinning forces.
[22] Brass A and Jensen H J 1989 *Phys. Rev.* B **44** 10 206
[23] The values of the fields in the paper correspond to a model relation $H_{c2} = H_{c2}(0)(1 - t^2)$ with $H_{c2}(0) = 127$ T. Practically, they may be smaller.